ChE-312 Numerical methods

Part 2

Exercise 7 solution.

7.

$$-\frac{d^2u}{dx^2} + u = 1 \qquad u(0) = 0 \qquad u(1) = 0$$

calculate the column vector $\mathbb{F}$ using the same Ω_k and ϕ_j from the lecture

$$\mathbb{F} = \sum_{k=1}^{4} \mathbb{F}^{(k)}$$

$$\mathbb{F}^{(k)} = \begin{bmatrix} \mathbb{F}_1^{(k)} \\ \mathbb{F}_2^{(k)} \\ \mathbb{F}_3^{(k)} \end{bmatrix}$$

$$\mathbb{F}_j^{(k)} = \int_{\Omega_k} 1 \cdot \phi_j(x)\, dx$$

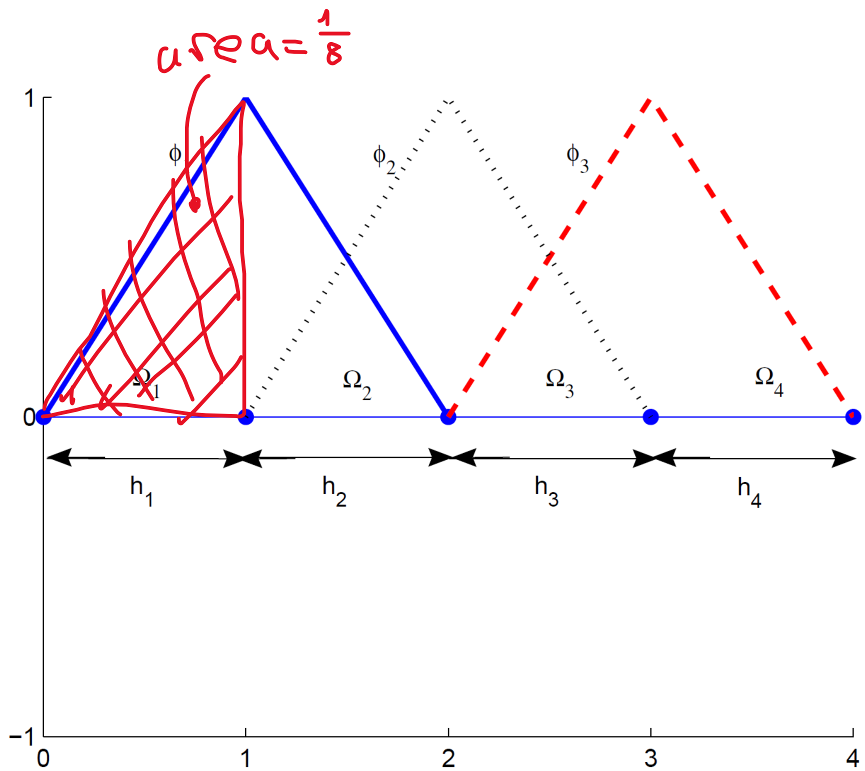
$$\mathbb{F}^{(1)} = \begin{bmatrix} \int_0^{1/4} 4x\, dx \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/8 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{F}^{(2)} = \begin{bmatrix} \int_{1/4}^{1/2} 4\left(\frac{1}{2} - x\right) dx \\ \int_{1/4}^{1/2} 4\left(x - \frac{1}{4}\right) dx \\ 0 \end{bmatrix} = \begin{bmatrix} 1/8 \\ 1/8 \\ 0 \end{bmatrix}$$

$$\mathbb{F}^{(3)} = \begin{bmatrix} 0 \\ \int_{1/2}^{3/4} 4\left(\frac{3}{4} - x\right) dx \\ \int_{1/2}^{3/4} 4\left(x - \frac{1}{2}\right) dx \end{bmatrix} = \begin{bmatrix} 0 \\ 1/8 \\ 1/8 \end{bmatrix}$$

$$\mathbb{F}^{(4)} = \begin{bmatrix} 0 \\ 0 \\ \int_{3/4}^{1} 4(1 - x)\, dx \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/8 \end{bmatrix}$$

Note all of the integrals come out to be 1/8, which can also be seen by geometrical inspection.



Then $\mathbb{F} = \mathbb{F}^{(1)} + \mathbb{F}^{(2)} + \mathbb{F}^{(3)} + \mathbb{F}^{(4)}$

$$\mathbb{F} = \begin{bmatrix} 1/8 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/8 \\ 1/8 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/8 \\ 1/8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/8 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$